HAVE INTERSTELLAR CLOUDS DISRUPTED THE OORT COMET CLOUD?

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ABSTRACT

We derive formulas describing the disruption rate of the Oort comet cloud due to encounters with interstellar clouds and field stars. For a comet with semimajor axis $a = 25\,000$ AU, the half-life due to encounters with stars is 3×10^9 yr, due to encounters with molecular clouds 3×10^9 yr, and due to encounters with atomic clouds 5×10^{10} yr. These results are based on a local density of molecular gas $\rho_0 = 0.024\ M_\odot/\mathrm{pc}^3$, a mean column density $N_\mathrm{H} = 10^{22}\ \mathrm{cm}^{-2}$, and a clumpiness factor of 2. We also assume that the mean density of molecular gas averaged over the solar lifetime was a factor of 1.5 higher than the present density. Thus it appears that molecular clouds have had a substantial, but not devastating, effect on the Oort cloud. However, many of the important parameters are rather uncertain, and the best argument that the Oort cloud has survived encounters with interstellar clouds is an observational one: a significant fraction of field stars are found in wide binary systems whose half-life due to encounters with interstellar clouds is within a factor of 2 of the half-life of the comets.

I. INTRODUCTION

For many years, it has been believed that the solar system is surrounded by a spherical distribution of comets (the "Oort cloud"; Oort 1950), in bound orbits with semimajor axes around 25 000 AU (e.g., Marsden, Sekanina, and Everhart 1978; this is the average value for new comets with large perihelion values, which are affected least by nongravitational effects). Encounters with passing interstellar clouds (IC's) can lead to the escape of comets from the Oort cloud, an effect first pointed out by Biermann and Lüst (1978) and Biermann (1978). In later work, Clube and Napier (1982), Napier and Staniucha (1982), and van den Bergh (1982) have taken the more extreme view that the Oort cloud is "decimated" by encounters with giant molecular clouds and hence that it cannot be primordial. These authors suggest that the Oort cloud must be continually replenished, either by scattering comets out of a reservoir at much smaller radii ($a \lesssim 10\,000$ AU) or by capture of comets from interstellar space. In a more careful analysis, Bailey (1983) concluded that it was "unlikely" that the Oort cloud could survive.

The disruption of the Oort cloud is difficult to reconcile with the high frequency among common G stars of wide binary stars with projected separations $\Delta r \approx 0.1$ pc $\approx 20\,000$ AU (Bahcall and Soneira 1981; Latham, Tonry, Bahcall, Soneira, and Schechter 1984). Although the binary stars have somewhat different properties from the Sun—in particular, they generally have large velocities perpendicular to the Galactic plane and hence spend less time in the IC layer—it is still somewhat surprising that they can survive if the comets cannot. In any event, the most natural picture is that the comet cloud formed at the time of formation of the solar system when the density of interplanetary gas and dust was high.

In this paper, we reinvestigate the disruptive effects of IC's on the Oort cloud, using a formula for the disruption of wide binary stars that was developed by Bahcall, Hut, and Tremaine (1985, hereafter referred to as Paper I).

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II. FORMULA FOR THE DISRUPTION RATE

In Paper I, we showed that the half-life of a binary star subject to encounters with pointlike field objects of mass M is given by

$$t_{1/2} = t_\mathrm{crit}\frac{M_\mathrm{crit}}{M}, \quad M \ll M_\mathrm{crit}$$
$$= t_\mathrm{crit}, \quad M \gg M_\mathrm{crit}. \tag{1}$$

In this formula,

$$t_\mathrm{crit} = \gamma\frac{M_\mathrm{crit}}{\rho V a^2}, \tag{2}$$

where γ is a dimensionless constant of order unity, ρ is the mass density of field objects, V is the rms velocity of the field objects relative to the binary, a is the semimajor axis of the binary, and

$$M_\mathrm{crit} = \beta\frac{(m_1+m_2)^{1/2}a^{1/2}V}{G^{1/2}} = \beta\frac{(M_1+m_2)V}{v}. \tag{3}$$

Here m_1, m_2 are the masses of the binary components, $v^2 = G(m_1+m_2)/a$ is the mean-square relative velocity of the binary components, and β is a dimensionless constant of order unity. These equations are based on the assumption that $v \ll V$ (the binary is "soft" in the terminology of Heggie 1975). We have calibrated the constants β and γ by numerical experiments and find $\beta = 0.023 \pm 0.005$, $\gamma = 2.2 \pm 0.5$ (see Paper I).

The small mass limit $M \ll M_\mathrm{crit}$ corresponds to close encounters between a field object and one of the binary members, with an impact parameter much smaller than the semimajor axis of the binary orbit. In this limit, the binary is torn apart gradually by the cumulative effect of many weak encounters which cause small changes in the binding energy (cf. Figure 1a of Paper I). The large mass limit $M \gg M_\mathrm{crit}$ corresponds to tidal encounters in which a field object passes at a distance much larger than the binary separation. In the large mass limit, the binary is generally disrupted by one or two strong encounters (Figure 1b of Paper I).

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Now let us specialize to the case where the binary consists of the Sun and a comet, and the field object is an IC. Of course, the IC's have finite sizes and a complicated clumpy structure, whereas our formulas apply only when the field objects are point masses. We shall discuss this issue further below, but it is worth noting here that the finite sizes of the IC's can only *reduce* their perturbing effects. Hence formulas based on point-mass perturbers will yield lower limits to the half-life.

We take $m_1 = M_\odot$, $m_2 = 0$, $a = 25\,000$ AU. Thus $v = (GM_\odot/a)^{1/2} = 0.19$ km s^{-1}. Let the rms relative velocity be $V = 22$ km s^{-1} (we justify this value later in the paper). From equation (3) we find

$$M_{\rm crit} = 2.7\, M_\odot. \tag{4}$$

Thus any reasonable cloud mass M satisfies $M \gg M_{\rm crit}$, and we may write

$$t_{1/2} = \beta\gamma \frac{(m_1+m_2)^{1/2}}{G^{1/2}\rho a^{3/2}} = \beta\gamma \frac{M_\odot^{1/2}}{G^{1/2}\rho a^{3/2}}, \tag{5}$$

where $\beta\gamma = 0.050 \pm 0.016$. This formula is originally due to Chandrasekhar (1944), who also analytically derived a very similar numerical coefficient $\beta\gamma = 0.080$. However, Chandrasekhar apparently did not recognize that the formula only applied for $M \gg M_{\rm crit}$.

The most striking feature of equation (5) is that the half-life is independent of both the relative velocity V and the cloud mass M (so long as $M \gg M_{\rm crit}$). This is an enormous simplification since the masses of the IC's are very poorly known, and since the clumpy nature of the IC distribution makes even the definition of the mass of an IC problematical. The only property of the IC's that we need to know is the average density ρ.

For a comet at $a = 25\,000$ AU, we may write equation (5) as

$$t_{1/2} = 1.8 \times 10^7 \text{ yr} \left(\frac{M_\odot/\text{pc}^3}{\rho}\right). \tag{6}$$

For our purposes, it is convenient to rewrite this result in the more general form

$$t_{1/2}^{-1} = (1.8 \times 10^7 \text{ yr})^{-1} f_z f_e f_t f_s f_p f_g \left(\frac{\rho_0}{M_\odot/\text{pc}^3}\right), \tag{7}$$

where ρ_0 is the density of IC's in the midplane of the Galaxy at the solar radius. The dimensionless efficiency factors f_z, f_e, f_t, f_s, f_p, and f_g represent modifications to the half-life due to the following effects:

(1) f_z: The Sun's vertical motion carries it away from the Galactic midplane to heights where the density of IC's is lower than the midplane. The factor f_z is the ratio of the disruption rate to the disruption rate which would obtain if the Sun remained exactly in the midplane.

(2) f_e: The Sun's epicyclic motion carries it through a range of Galactocentric radii. The factor f_e is the ratio of the disruption rate to the disruption rate which would obtain if the Sun always remained at its present Galactocentric radius.

(3) f_t: The half-life formula (5) is based on the impulse approximation, which is the approximation that the encounter time during which the IC passes by the Sun is small compared to the orbital time of the comet around the Sun (4×10^6 yr for $a = 25\,000$ AU). In some encounters of interest, this approximation may fail. The factor f_t is the ratio of the disruption rate to the disruption rate which would obtain if the impulse approximation were valid for all encounters.

(4) f_s: The IC's have finite size. The factor f_s is the ratio of the disruption rate to the disruption rate which would obtain if the IC's were collapsed into point masses.

(5) f_p: The density of IC's was probably higher in the past. The factor f_p is the ratio of the disruption rate to the disruption rate which we estimate by assuming that the density of IC's has been independent of time.

(6) f_g: The encounter rate between IC's and the solar system is enhanced by gravitational focusing, an effect neglected in deriving equation (6). The factor f_g is the increase in the disruption rate due to the fact that the relative orbit of the IC and the solar system is a hyperbola rather than a straight line.

III. EVALUATION OF THE EFFICIENCY FACTORS

There are two main types of IC's: atomic or H I clouds and molecular or H_2 clouds. Since the properties of these two types of IC are very different, we shall consider their disruptive effects separately.

a) Molecular Clouds

We can write the density of molecular clouds (MC's) in the solar neighborhood as

$$\rho(z) = \rho_0 e^{-z^2 \ln 2/z_{1/2}^2}, \tag{8}$$

where z is the distance above the midplane and $z_{1/2}$ is the half-thickness of the MC's. Sanders, Solomon, and Scoville (1984) estimate that the density and half-thickness of molecular gas at the solar radius are $\rho_0 = 0.033\ M_\odot/\text{pc}^3$ and $z_{1/2} = 75$ pc. One major uncertainty in ρ_0 is in the conversion factor from the integrated CO line intensity to column density of H_2, which they take to be $N(H_2)/\int T_A(\text{CO})\,dv = 3.6 \times 10^{20}$ molecules cm^{-2} K^{-1} (km s^{-1})$^{-1}$. Bloemen *et al.* (1984) use gamma-ray observations by the *COS-B* satellite to provide an independent determination of this conversion factor, and arrive at a value lower than that used by Sanders *et al.* by a factor $2.6/3.6 = 0.7$, which would lead to a density of molecular gas $\rho_0 = 0.024\ M_\odot/\text{pc}^3$. Dame and Thaddeus (1985) find an even lower value, $\rho_0 = 0.013\ M_\odot/\text{pc}^3$. We will adopt the intermediate value $\rho_0 = 0.024\ M_\odot/\text{pc}^3$ in the following discussion; a higher or lower choice for the density simply scales to a proportionately higher or lower destruction rate of comets.* Thus equation (7) becomes

$$t_{1/2}^{-1} = (7.5 \times 10^8 \text{ yr})^{-1} f_z f_e f_t f_s f_p f_g. \tag{9}$$

We shall now estimate the efficiency factors.

1) The z-mode factor f_z

The proper value to use for the MC density in equations for the half-life is the time-averaged density of MC's in the vicinity of the Sun. This average is smaller than ρ_0 because the vertical motion of the Sun regularly carries it partway out of the cloud layer. The factor f_z is simply the ratio of this time-averaged density to ρ_0. To determine f_z, we must first know the Sun's vertical orbit. We may compute the present solar orbit by assuming that the total density in the Galactic disk $\rho_{\rm tot}$ is independent of z. This should be a good approxi-

*Note that an upper limit $\rho_0 < 0.17\ M_\odot/\text{pc}^3$ is given by the Oort limit to the total density (Bahcall 1984a,b).

mation because most of the mass probably resides in an old population with a large half-thickness. With this assumption, the potential is that of an harmonic oscillator, $\Phi(z) = 2\pi G\rho_{\rm tot} z^2$, and the Sun's vertical energy $E_{z\odot} = \frac{1}{2} v_z^2 + \Phi(z)$ is conserved. Using equation (8) for the density distribution of IC's, it is then straightforward to show that

$$f_z = e^{-x} I_0(x), \tag{10}$$

where $I_0(x)$ is a modified Bessel function and

$$x = \frac{E_{z\odot} \ln 2}{4\pi G\rho_{\rm tot}\, z_{1/2}^2}. \tag{11}$$

This result is given in integral form as equation (55) of Bailey (1983). Since the Sun is presently close to the Galactic midplane, we may take $E_{z\odot} = \frac{1}{2} v_z^2$, where $v_z = 7$ km s^{-1} is our present z velocity relative to the local standard of rest. For future use, we also define the rms z velocity of the Sun $\sigma_{z\odot}$; because the time-averaged kinetic energy is equal to half the total energy in a quadratic potential, we have

$$\sigma_{z\odot}^2 = E_{z\odot}, \tag{12}$$

and thus $\sigma_{z\odot} = 7$ km $^{-1}/\sqrt{2} = 5$ km s^{-1} at the present epoch. Bahcall (1984a) gives $\rho_{\rm tot} = 0.185 \pm 0.02\, M_\odot/{\rm pc}^3$ and we shall take $z_{1/2} = 75$ pc following the estimate of Sanders *et al.* (1984). With these parameters, equations (10) and (11) yield $x = 0.30$ and $f_z = 0.76$ for the present solar orbit.

However, this estimate does not account for the diffusion of the Sun in velocity space over its lifetime. There is considerable evidence that disk stars are born with very small z velocities, and that their z velocities gradually increase by a diffusion process or random walk in velocity space, perhaps due to encounters with MC's or transient spiral arms (Wielen 1977). Thus we must consider the solar z energy and rms velocity to be functions of time $E_{z\odot}(t)$, $\sigma_{z\odot}(t)$, where $\sigma_{z\odot}(t)$ is understood to refer to an average over a time interval long compared to the orbit period but short compared to the age of the Sun. Notice that $E_{z\odot}(t=0) = 0$ and $\sigma_{z\odot}(t=0) = 0$, since we assume that the Sun is born with a small peculiar velocity at $t=0$, and note that equation (12) still holds at each time t. At the present epoch t_0, the rms z velocity of G0 dwarfs like the Sun is $\sigma_{z,{\rm G0}}(t_0) = 20$ km s^{-1} (Mihalas and Binney 1981, p. 423), which is quite large compared to $\sigma_{z\odot}(t_0) = 5$ km s^{-1}. Thus the Sun's present low z velocity is a rather unusual chance event, and we should expect that its z velocity was higher in the past. We must therefore attempt to estimate the Sun's rms z velocity averaged over its entire lifetime. To obtain a crude estimate, we may use a simple model in which the diffusion rate is independent of both time and the velocity of the star.* In this model, the mean-square z velocity of a group of stars increases linearly with time. Hence the rms z velocity of the G0 dwarfs, averaged over their lifetime, is simply

$$\langle \sigma_{z,{\rm G0}}^2(t)\rangle^{1/2} = \sigma_{z,{\rm G0}}(t_0)/\sqrt{2} = 14 \text{ km s}^{-1}. \tag{13}$$

In the absence of other information, this would be the best estimate for the Sun's lifetime averaged rms z velocity. However, we also know the present value of the Sun's rms z velocity. Thus we should only consider those paths in the random walk which end at the present value of the rms z velocity $\sigma_{z\odot}(t_0)$. From the theory of Brownian motion in a quadratic potential (e.g., Chandrasekhar 1943), it can be shown that, with this constraint, the best estimate of the mean-square z velocity at time t is

$$\sigma_{z\odot}^2(t) = \sigma_{z,{\rm G0}}^2(t_0)\frac{t(t_0-t)}{t_0^2} + \sigma_{z\odot}^2(t_0)\frac{t^2}{t_0^2}, \tag{14}$$

where we have assumed that the Sun and all the other G0 stars were born at time $t=0$. Thus, averaging over t, we find the best estimate for the Sun's lifetime average rms z velocity,

$$\langle \sigma_{z\odot}^2(t)\rangle^{1/2} = \left[\tfrac{1}{6}\sigma_{z,{\rm G0}}^2(t_0) + \tfrac{1}{3}\sigma_{z\odot}^2(t_0)\right]^{1/2}. \tag{15}$$

Inserting $\sigma_{z,{\rm G0}}(t_0) = 20$ km s^{-1} and $\sigma_{z\odot}(t_0) = 5$ km s^{-1}, we find $\langle\sigma_{z\odot}^2(t_0)\rangle^{1/2} = 8.6$ km s^{-1}. The best estimate for the Sun's lifetime average z energy is thus $\langle E_{z\odot}(t)\rangle = (8.6$ km s$^{-1})^2$, which by equations (10) and (11) yields $x = 0.91$ and

$$f_z = 0.5, \tag{16}$$

the value we shall use henceforth.

2) The epicycle factor f_e†

The density of MC's in the vicinity of the Sun also depends on the distance of the Sun from the Galactic center. The proper value to use for ρ_0 in the half-life equation (7) is the average value of ρ_0 over one radial period of the solar orbit. Epicycle theory (e.g., Chandrasekhar 1942) shows that the mean or guiding center radius of the solar orbit R_e is related to the Sun's present radius R by

$$R_e - R = \frac{v_\phi - V_c}{2B}, \tag{17}$$

where B is one of Oort's constants and $v_\phi - V_c$ is the solar velocity relative to the local standard of rest in the direction of Galactic rotation. Using $R = 10$ kpc, $v_\phi - V_c = 12$ km s^{-1} (Mihalas and Binney 1981, p. 400), and $B = 11$ km s^{-1} kpc^{-1} (Mihalas and Binney 1981, p. 480) we find $R_e - R = 0.5$ kpc. Similar calculations show that the amplitude of the Sun's radial excursions is 0.6 kpc; that is, the Sun oscillates between about 9.9 and 11.1 kpc from the Galactic center, and is presently near the pericenter of its orbit. The surface density of molecular gas is a steeply falling function of Galactocentric radius near the Sun. If the decrease in surface density is reflected by a similar decrease in the midplane volume density (i.e., if the half-thickness remains roughly constant), then the time-averaged value of the midplane volume density of molecular gas is smaller than its value at the present solar radius. Observations of external galaxies generally show that the CO emission is proportional to the optical surface brightness (cf. Fig. 14 of Sanders *et al.* 1984); adopting an exponential disk model with a scale length of 4.4 kpc (from Bahcall and Soneira 1980, scaled to a solar radius of 10 kpc), we find that the surface density at the

*The assumption that the diffusion rate is independent of velocity has indirect observational support: a diffusion process of this sort produces a Gaussian distribution of z velocities, consistent with the shape of the observed distribution (Bahcall 1984b).

†The importance of this factor was pointed out to us by R. Wielen. Wielen has also stressed that diffusion of the Sun in velocity space can affect both the epicycle amplitude and the mean radius of the solar orbit, and that changes in either quantity can affect the time-averaged density of MC's in the solar vicinity. For example, Wielen (1977) finds that diffusion causes a slow outward drift of about 1 kpc over 10^{10} yr in his numerical simulations of Galactic orbits. We have not included these effects since they are likely to be small, and, in addition they are very difficult to disentangle from any possible evolution of the radial distribution of the MC's themselves.

mean radius R_e is smaller than at R by a factor 0.9. Observations by Sanders *et al.* (1984) suggest a more rapid falloff, with the surface density dropping by a factor of 5 between 10 and 11.5 kpc, in which case the surface density at R_e would be smaller than at R by about a factor of 0.5. As a compromise, we adopt

$$f_e = 0.7. \tag{18}$$

3) The finite-time factor f_t

The validity of the impulse approximation can be parametrized by the quantity

$$\chi = \frac{np}{V_0}, \tag{19}$$

where $n = (GM_\odot/a^3)^{1/2}$ is the mean motion of the comet, p is the impact parameter of the cloud, and V_0 is the relative velocity. For $\chi \ll 1$, the impulse approximation is valid. For $\chi \gg 1$, the actual energy change is reduced from the change predicted by the impulse approximation by a factor proportional to $\chi^{3/2}\exp(-\chi)$ (Yabushita 1972).* Thus for $\chi \gtrsim 1$ the impulse approximation seriously overestimates the energy change in an encounter.

To determine whether the failure of the impulse approximation affects our estimates of the half-life, we must estimate the impact parameters of the encounters which dominate the disruption rate. Assuming that the impulse approximation is valid and that binaries are disrupted in a single encounter, a straightforward but tedious calculation shows that the mean-square impact parameter of all encounters that cause disruption is

$$p_{av}^2 = \delta \frac{G^{1/2}Ma^{3/2}}{(m_1+m_2)^{1/2}V} = \beta\delta a^2\left(\frac{M}{M_{\rm crit}}\right), \tag{20}$$

where the second line follows from equation (3), and the dimensionless factor $\delta = 5{\cdot}3^{5/2}{\cdot}2^{-9/2}\pi^{-1/2} = 1.94$ if the distribution of relative velocities is Maxwellian.† Thus the typical value of χ in an encounter which disrupts comets is

$$\chi_{av} = \frac{np_{av}}{V} = 0.21\,\frac{v}{V}\left(\frac{M}{M_{\rm crit}}\right)^{1/2}, \tag{21}$$

where v is the rms velocity of the comets. Inserting $v = 0.19$ km s^{-1}, $V = 22$ km s^{-1} (to be derived below), and $M_{\rm crit} = 2.7\,M_\odot$ from Eq. (4), we find

$$\chi_{av} = 1.1\left(\frac{M}{10^6\,M_\odot}\right)^{1/2}. \tag{22}$$

Thus the impulse approximation should be accurate so long as $\chi_{av} \lesssim 1$, i.e., for $M \lesssim 10^6 M_\odot$. Solomon and Sanders (1979) estimate that MC's have a typical mass of $5\times10^5\,M_\odot$, with most MC masses lying in the range $10^5\,M_\odot$ to $10^6\,M_\odot$. Blitz (1979) quotes a mean mass of $10^5\,M_\odot$ with a range from $3\times10^4\,M_\odot$ to $3\times10^5\,M_\odot$. Thus the use of the impulse approximation should be safe, except perhaps for the most massive clouds. Even if the cloud masses have been underestimated, the error introduced by the impulse approximation should not be too severe, for two reasons. (1) χ_{av} represents the typical value of χ for the closest encounter expected in a time interval of about $t_{1/2}$ [see footnote following equation (20)]. If $\chi_{av} \gtrsim 1$, then the comets survive until an encounter occurs with $\chi \lesssim 1$, which requires a time interval longer by a factor χ_{av}^2. Thus the half-life is lengthened by a factor $\max(\chi_{av}^2, 1)$; and even if the appropriate MC mass were as large as, say, $2\times10^6\,M_\odot$, the impulse approximation would only underestimate the half-life by a factor of 2. (2) Our estimates of the half-life are based on the assumption that the MC's are mass points. In fact, the cloud radii can easily exceed p_{av} (as below), so that the most destructive encounters may involve penetration of the cloud by the solar system. In this case, the proper value of χ_{av} depends on whether the distribution of mass in the cloud is smooth or clumpy. If the mass distribution is smooth, we should replace p_{av} in equation (21) by the cloud radius r_c. Solomon and Sanders (1979) estimate that a cloud with mass $5\times10^5\,M_\odot$ has $r_c = 20$ pc, so that $\chi_{av} = 1.4$, in which case the impulse approximation could overestimate the disruption rate by a factor of 2.5 for homogeneous clouds. However, it appears that the distribution of mass in the clouds is clumpy, and in this case the proper value of χ_{av} for a penetrating encounter is determined by the mass and radius of an individual clump; these will be much smaller than the mass and radius of the whole MC and hence on the whole the impulse approximation should be quite good.

In view of these considerations, the most reasonable procedure is to assume that the impulse approximation is always valid, and thus to take

$$f_t = 1. \tag{23}$$

At this point, we pause to derive the value of the rms relative velocity V, which we have used in equations (4) and (22). We first find the Sun's lifetime average rms velocity $\langle\sigma_{{\rm tot},\odot}^2(t)\rangle^{1/2}$. Using arguments similar to those leading to equation (15), we have

$$\langle\sigma_{{\rm tot},\odot}^2(t)\rangle^{1/2} = \left[\tfrac{1}{6}\,\sigma_{\rm tot,G0}^2(t_0) + \tfrac{1}{3}\,\sigma_{{\rm tot},\odot}^2(t_0)\right]^{1/2}, \tag{24}$$

where $\sigma_{\rm tot,G0}(t_0) = 37$ km s^{-1} is the present rms velocity of G0 dwarfs relative to the local standard of rest (Mihalas and Binney 1981, p. 423), and $\sigma_{{\rm tot},\odot}(t_0) = 17$ km s^{-1} is the present rms velocity of the Sun relative to the local standard of rest, obtained using the epicyclic orbit discussed after equation (17). Thus the lifetime average rms velocity of the Sun is

$$\langle\sigma_{{\rm tot},\odot}^2(t)\rangle^{1/2} = 18 \text{ km s}^{-1}. \tag{25}$$

We next compute the rms velocity of the MC's. Their rms z velocity $\sigma_{z,\rm MC}$ is related to the half-thickness $z_{1/2}$ by

$$\sigma_{z,\rm MC} = z_{1/2}\sqrt{\frac{2\pi G\rho_{\rm tot}}{\ln 2}}\,; \tag{26}$$

adopting $z_{1/2} = 75$ pc and $\rho_{\rm tot} = 0.185\ M_\odot/{\rm pc}^3$ we find $\sigma_{z,\rm MC} = 6.4$ km s^{-1}. The ratio of the rms z velocity to the rms total velocity depends on the shape of the velocity ellipsoid. If the ellipsoid is spherical, $\sigma_{\rm tot,MC} = \sqrt{3}\sigma_{z,\rm IC} = 11$

*For precisely circular orbits, the reduction factor is proportional to $\chi^{3/2}\exp(-2\chi)$ (Spitzer 1958).

† An approximate version of this result can be derived by a simple heuristic argument. Since the maximum tidal force from a passing MC falls rapidly with increasing impact parameter p, the disruption rate is strongly dominated by the encounters with the smallest impact parameters. In one half-life $t_{1/2}$, the number of encounters with impact parameters less than p is $N(p,t_{1/2}) = (8\pi/3)^{1/2}p^2(\rho/M)Vt_{1/2}$. Setting $N(p,t_{1/2}) = 1$ and evaluating $t_{1/2}$ using equation (5), we find that the impact parameter of the closest encounter is given by $p^2 = 6.9G^{1/2}Ma^{3/2}/[V(m_1+m_2)^{1/2}]$, which is the same as (20) except for the numerical factor.

km s^{-1}; if the ellipsoid has the same shape as the stellar velocity ellipsoid (Wielen 1977) then $\sigma_{\text{tot,MC}} = 2.4\sigma_{z,\text{MC}} = 15$ km s^{-1}. We shall use a compromise value

$$\sigma_{\text{tot,MC}} = 13 \text{ km s}^{-1}. \tag{27}$$

For want of a better assumption, we shall assume that $\sigma_{\text{tot,MC}}$ is independent of time. Hence the best estimate for the rms relative velocity of the Sun and a MC is

$$V = [\sigma^2_{\text{tot,MC}} + \langle \sigma^2_{\text{tot},\odot}(t) \rangle]^{1/2} = 22 \text{ km s}^{-1}, \tag{28}$$

the value that we use in equations (4) and (22).

4) The finite-size factor f_s

According to equations (20) and (4), the rms impact parameter of the encounters which disrupt a comet at $a = 25\,000$ AU is

$$p_{av} = 0.016 \text{ pc} \left(\frac{M}{M_\odot}\right)^{1/2}. \tag{29}$$

If p_{av} is smaller than the cloud radius r_c, then the disruptive effect of the clouds will be reduced. In terms of the mean surface density of the cloud, $\Sigma = M/\pi r_c^2$, $p_{av} < r_c$ if $\Sigma \lesssim 1.2 \times 10^3 M_\odot$ pc^{-2}, or, in terms of the column density of hydrogen, if $N_{\text{H}} \lesssim 1 \times 10^{23}$ cm^{-2}. Solomon and Sanders (1979) estimate that their typical cloud has $N_{\text{H}} = 3 \times 10^{22}$ cm^{-2}. Rowan-Robinson (1979) finds that the mean column density of MC's is about $N_{\text{H}} = 6 \times 10^{21}$ cm^{-2} with a spread of a factor 2–3, and Spitzer (1978) quotes a similar value. Thus the approximation that the clouds are point masses may not be valid.

To analyze this effect, we consider the limit where the IC's are very diffuse, so that the maximum velocity perturbation received by a binary during a passage through a cloud is much less than the escape speed. We show in the Appendix that in this case the half-life is

$$t_{1/2} = 0.0025 \frac{V(m_1 + m_2)}{G\rho a^3 \Sigma_{av} C}, \tag{30}$$

where Σ_{av} is the mean surface density of a cloud and the clumping factor C is given by

$$C = \frac{A \int_A \Sigma^2 dA}{(\int \Sigma dA)^2}, \tag{31}$$

where Σ is the cloud surface density and the integral is over the area of the cloud.

One can get a feeling for the magnitude of the dimensionless clumping factor C by considering two extreme cases. In the first case, we have a complete lack of clumping, which defines a homogeneous cloud, for which $C = 9/8$, slightly larger than unity, because even for a homogeneous cloud the column density is not constant but falls off from center to edge. In the second case, we introduce strong clumping by dividing all the matter in the cloud over an arbitrary number of identical homogeneous subclumps, which are small enough so that none of them overlap when projected on the sky. In this case, $C = \frac{9}{8} f_\Sigma^{-1}$, where f_Σ is the area-filling factor, i.e., the fraction of the projected area on the sky covered by subclumps.

The finite-size factor follows from equations (5) and (30) as $f_s = \Sigma_{av} C / \Sigma_{\text{crit}}$, where

$$\Sigma_{\text{crit}} = 0.05 \frac{V(m_1 + m_2)^{1/2}}{G^{1/2} a^{3/2}}. \tag{32}$$

With our standard parameters $a = 25\,000$ AU, $V = 22$ km s^{-1}, $m_1 + m_2 = M_\odot$, we find $\Sigma_{\text{crit}} = 400\ M_\odot$ pc^{-2}, which corresponds to $N_{\text{H}} = 3.7 \times 10^{22}$ cm^{-2}. Since the disruption rate can never exceed the value for point masses, we may combine the calculations for the point-mass limit and the diffuse-cloud limit by the interpolation formula

$$f_s = \min\left(\frac{\Sigma_{av} C}{\Sigma_{\text{crit}}}, 1\right). \tag{33}$$

Our expression for the clumping factor C is only valid when the clouds are diffuse and will overestimate the disruptive effects if the surface density is high; to account for this, we should replace the surface density Σ in (31) by $\min(\Sigma, \Sigma_{\text{crit}})$.

The uncertainty in the disruption rate resulting from the diffuse nature of the MC's is probably the most serious uncertainty in the calculations of this paper, both because the calibration of the mean surface density is uncertain, and because the degree of clumpiness in the clouds is poorly known. Our best estimate is that the mean column densities are typically lower than Σ_{crit} by a factor of 5 or so, leading to $f_s = 0.2$, but that clumpiness increases the value of f_s by a factor of 2 or 3. Values of f_s close to unity seem unlikely since this requires that *all* of the molecular gas is in regions with $\Sigma \gtrsim \Sigma_{\text{crit}}$. Thus, we take

$$f_s = 0.5, \tag{34}$$

with the understanding that f_s may lie anywhere in the range $1 \geqslant f_s \gtrsim 0.1$.

5) The different-past factor f_p

To estimate this factor, we assume that the half-thickness of the MC layer and the fraction of gas in MC's are independent of time. Thus f_p is simply the ratio of the average gas surface density over the past 4.5×10^9 yr to the present gas surface density. This ratio can be estimated from evolutionary models of the Galactic disk (see Tinsley 1980 for a review). Ostriker and Thuan's (1975) models give $f_p = 1.4$, while Twarog's (1980) model gives $f_p = 2.6$. In general, a relatively small value of f_p, like Ostriker and Thuan's, is preferable for two reasons: (1) Twarog's analysis of the observations shows that the star-formation rate has remained relatively constant over the past 4×10^9 yr, and this result would be surprising if the gas surface density changed by a large factor over that period. (2) The midpoint of the Sun's lifetime was only 2.3×10^9 yr ago, which is only about 15% of the age of the Galaxy, and it seems unreasonable that the gas surface density should change by as much as a factor of 2 over such a short interval.* Thus we feel that $f_p \approx 2$ is an upper limit, and we adopt

$$f_p = 1.5, \tag{35}$$

with the understanding that f_p could lie anywhere in the range $1 \lesssim f_p \lesssim 2$.

6) The gravitational focusing factor f_g

The principal effect of gravitational focusing is to enhance the collision rate between the solar system and the MC's. If we approximate the distribution of relative velocities as an isotropic Maxwellian with rms velocity $V = 22$ km s^{-1}, then

*Thus, for example, Twarog's model predicts that all of the interstellar gas will be converted to stars in less than 1.5×10^9 yr, a very short time compared with the age of the Galaxy.

the collision rate with clouds of mass M and radius r_c is increased by a factor*

$$1+\frac{3G(M+M_\odot)}{V^2 r_c}\equiv 1+\left(\frac{M}{M_g}\frac{N_H}{10^{23}\ \mathrm{cm}^{-2}}\right)^{1/2}, \quad (36)$$

where N_H is the mean column density per cloud and

$$M_g = 4\times 10^5\, M_\odot.$$

Thus, for a typical cloud with $N_H \approx 10^{22}$ cm^{-2} (see Subsec. *a4)*, the enhancement of the collision rate is generally quite small; even if $M = 10^6\, M_\odot$, the enhancement is only a factor of 1.5. This result shows that for most encounters, the effect of gravitational focusing is quite small, and we can safely take

$$f_g = 1. \quad (37)$$

b) Atomic Clouds

The mean H I density in the Galactic plane is 0.7 cm^{-3} (Spitzer 1978, hereafter referred to as S78, Sec. 3.3b), corresponding to $\rho_0 = 0.024\, M_\odot/\mathrm{pc}^3$. Thus equation (7) yields

$$t_{1/2}^{-1} = (7.5\times 10^8\ \mathrm{yr})^{-1} f_z f_e f_t f_s f_p f_g. \quad (38)$$

1) The z-motion factor f_z

The half-thickness of the H I layer is $z_{1/2} = 120$ pc according to S78 (Sec. 11.1a); thus, using our value for the Sun's lifetime average z energy $\langle E_{z\odot}(t)\rangle = (8.6\ \mathrm{km\ s}^{-1})^2$ and equation (11), we find $x = 0.35$ and

$$f_z = 0.7. \quad (39)$$

2) The epicycle factor f_e

The midplane density of H I is almost independent of Galactocentric radius between 7 and 11 kpc (S78, Sec. 3.3b); hence

$$f_e = 1. \quad (40)$$

3) The finite-time factor f_t

The mass of the "standard" H I cloud is about 400 $M_\odot$ (S78, Table 7.2) so that $\chi_{av} \ll 1$ [equation (22)], and

$$f_t = 1. \quad (41)$$

4) The finite-size factor f_s

Column densities of H I clouds show a large spread. The standard H I cloud has $N_H = 4\times 10^{20}$ cm^{-2} (S78, Sec. 7.2a), while the rarer "large" clouds have $N_H = 1.7\times 10^{21}$ cm^{-2} (S78, Table 7.1), and the smallest clouds ("cloudlets") seen in 21-cm emission have $N_H = 2\times 10^{19}$ cm^{-2} (S78 Sec. 3.3b). The column densities are on average significantly smaller than those of molecular clouds. This is not surprising, since the fraction of H_2 in a cloud can be expected to be larger in a denser cloud, where formation of H_2 molecules (on grain surfaces) is more efficient (cf. Shull and Beckwith 1982). Savage, Bohlin, Drake, and Budich (1977) find in a survey with the *Copernicus* ultraviolet telescope that the fraction of molecular hydrogen in interstellar clouds undergoes a transition from low to high values around $E(B-V)\approx 0.08$, corresponding to $N_H \sim 5\times 10^{20}$ cm^{-2}. This does not mean that *no* dominantly atomic clouds exist at somewhat higher column densities (cf. Fig. 6 in Savage *et al.*), but it does imply that in *typical* H I clouds the surface density is not much larger than $N_H \sim 10^{21}$ cm^{-2}. The clouds do not appear to be very clumpy. Thus we choose

$$f_s = 0.014, \quad (42)$$

with a firm upper limit $f_s \lesssim 0.03$.

5) The different-past factor f_p

We use the same different-past factor for atomic and molecular gas [Eq. (35)],

$$f_p = 1.5. \quad (43)$$

6) The gravitational focusing factor f_g

Gravitational focusing is unimportant for atomic hydrogen clouds, which are less massive and have lower escape speed than molecular clouds [cf. Eq. (36)], and therefore

$$f_g = 1. \quad (44)$$

IV. RESULTS FOR THE DISRUPTION RATE, AND COMPARISON WITH OTHER AUTHORS

For the half-life of comets ($a = 25\,000$ AU) due to encounters with molecular clouds, we find

$$t_{1/2}^{-1} = (1.8\times 10^7\ \mathrm{yr})^{-1} f_z f_e f_t f_s f_p f_g \left(\frac{\rho_0}{M_\odot/\mathrm{pc}^3}\right)$$
$$= (1.8\times 10^7\ \mathrm{yr})^{-1}\times 0.5\times 0.7\times 1\times 0.5\times 1.5\times 1\times 0.024 = (2.8\times 10^9\ \mathrm{yr})^{-1}. \quad (45)$$

For the half-life due to encounters with atomic clouds, we find

$$t_{1/2}^{-1} = (1.8\times 10^7\ \mathrm{yr})^{-1} f_z f_e f_t f_s f_p f_g \left(\frac{\rho_0}{M_\odot/\mathrm{pc}^3}\right)$$
$$= (1.8\times 10^7\ \mathrm{yr})^{-1}\times 0.7\times 1\times 1\times 0.014\times 1.5\times 1\times 0.024 = (5.0\times 10^{10}\ \mathrm{yr})^{-1}. \quad (46)$$

Adding reaction rates gives a combined half-life

$$t_{1/2}^{-1} = (2.8\times 10^9\ \mathrm{yr})^{-1} + (5.0\times 10^{10}\ \mathrm{yr})^{-1} = (2.7\times 10^9\ \mathrm{yr})^{-1}, \quad (47)$$

*The analogous equation (2) in Clube and Napier (1982) and equation (2) in Napier and Staniucha (1982) both are incorrect, apparently because in the Maxwellian they leave out the factor v^2 which arises from the velocity volume element $d^3v = 4\pi v^2 dv$.

or roughly 60% the age of the solar system.

The principal uncertainties in this result are in the finite size factor f_s and in the mean molecular density ρ_0. Even in the extreme case where $f_s = 1$, the Oort cloud would be completely disrupted (say, $t_{1/2}$ less than $\frac{1}{5}$ of the age of the solar system, so that only $2^{-5} = 0.03$ of the original comets remain) only if $\rho_0 \gtrsim 0.038\ M_\odot/\text{pc}^3$, somewhat beyond the range of values quoted in Sec. III*a*. However, the results are strongly dependent on the relatively uncertain conversion from CO intensity to column density of H_2, which affects the half-life both through the mean density ρ_0 and through the efficiency factor f_s. A factor of 2 decrease in the conversion factor yields a factor of 4 increase in the half-life, although a factor of 2 increase yields less than a factor of 4 decrease in the half-life since f_s is already close to unity. Thus our best estimate is that IC's have had a substantial—but not devastating—effect on the Oort cloud, removing perhaps two-thirds of the cloud at $a = 25\ 000$ AU.

It is interesting to compare this result with the half-life of the Oort cloud due to encounters with field stars, which can be estimated using equation (1) in the limit $M \ll M_{\text{crit}}$. We replace ρM by $\int_{M_{\min}}^{\infty} n(M)\, M^2\, dM$, where $n(M)$ is the number density of stars with mass M. Using parameters from Bahcall and Soneira (1980), and taking $M_{\min}$ to be the minimum mass for hydrogen burning ($M_{\min} \approx 0.085\ M_\odot$), we find $\rho M = 0.030\ M_\odot{}^2/\text{pc}^3$. We estimate V by adding in quadrature the Sun's lifetime average rms velocity of 18 km/s [equation (25)] and the rms velocity of local stars of 40 km/s. For comets at $a = 25\ 000$ AU, the half-life due to stellar encounters is then $t_{1/2} = 3 \times 10^9$ yr, almost the same as our best estimate of the half-life due to IC's. Thus IC's appear to be about as effective as stars in pruning the cloud, although the uncertainties in the cloud estimate are large.

Biermann and Lüst (1978) were the first to call attention to the potentially damaging effects of molecular clouds on the Oort cloud. They gave an example of a very dense and very heavy molecular cloud, and showed that a passage of the Sun near such a cloud would significantly deplete the comet cloud. The first quantitative analysis of the disruption rate was due to Biermann (1978), who found that encounters with IC's would seriously deplete the Oort cloud at semimajor axes $a \gtrsim 25\ 000$ AU. He concluded that the Oort comet cloud hypothesis was viable, but that the outer radius of the comet cloud was somewhat smaller than originally envisaged by Oort (1950). We basically agree with Biermann's (1978) analysis, and with his conclusion that a primordial comet cloud can survive passages with interstellar clouds. Our results can be compared with the help of equation (7). Biermann starts with a density of $\rho_0 = 0.028\ M_\odot/\text{pc}^3$, similar to our value $\rho_0 = 0.024\ M_\odot/\text{pc}^3$. He does not take into account the vertical or epicyclic motion of the Sun, and thereby effectively uses $f_z = f_e = 1$. He also neglects the time evolution of the gas density, setting $f_p = 1$. Like us, he sets $f_t = f_g = 1$. For a typical cloud, he takes $M = 10^{3.5}\ M_\odot$ and $r_c = 3$ pc, which implies $N_H = 1 \times 10^{22}$ cm^{-2} $= 0.27 N_{\text{crit}}$ and therefore $f_s = 0.3$ With these values, equation (7) would give $t_{1/2} = 2.3 \times 10^9$ yr. This result is roughly consistent with Biermann's conclusion that comets with $a = 25\ 000$ AU are disrupted in about the lifetime of the solar system, although Biermann's formulas appear to underestimate the disruption rate by about a factor of 2 relative to the more accurate equation (30).

Napier and Staniucha (1982) and van den Bergh (1982) obtained quite different results from those found by Biermann. Their results implied that all but the innermost comets ($a \lesssim 10^3$ AU) would have been lost through encounters with MC's. Why are their results so different from ours? First of all, van den Bergh's analysis is simply incorrect, since he computes the tidal radius that would be imposed on the Oort cloud if the Sun were in a circular orbit around an MC. The disruptive effect of a hyperbolic encounter with a MC is much smaller.

Napier and Staniucha's work needs a more careful analysis. They consider a range of cloud masses, but concentrate on a typical cloud mass of $5 \times 10^5\ M_\odot$, which results in an average density of molecular cloud material $\rho_0 = 0.027\ M_\odot/\text{pc}^3$, only slightly larger than our value $\rho_0 = 0.024\ M_\odot/\text{pc}^3$. However, most of their efficiency factors are very different from ours. (1): Their z-motion factor f_z is unity, since they do not correct for this effect. Their adopted half-thickness is 50 pc, instead of our 75 pc, which would lead to the value $f_z = 0.3$, if the z motion of the Sun out of the MC layer were properly taken into account using equation (10). (2): Their epicycle factor f_e is unity, since they do not correct for this effect. (3): Their finite-time factor f_t is unity, as is ours. (4): Their finite-size factor f_s is unity. They adopt values for a typical cloud of $M = 5 \times 10^5\ M_\odot$ and $r_c = 20$ pc. These values imply a column density $N_H = 3.6 \times 10^{22}$ cm^{-2} $\approx N_{\text{crit}}$, and therefore an efficiency factor $f_s = 1$, which is larger than our value $f_s = 0.5$. In addition, they assumed that *all* the mass was clumped in 25 equal subcondensations of $2 \times 10^4\ M_\odot$ each, with radii of 2 pc each. For each subcondensation, the column density $N_H = 1.4 \times 10^{23}$ cm^{-2} $= 4 N_{\text{crit}}$, far in excess of observed values. Consistent with their assumption that $N_H > N_{\text{crit}}$, they treat the subclumps as point masses, which implies that they used an effective value $f_s = 1$. (5): Their different-past factor $f_p = 1.5$, in agreement with our value. (6): Their gravitational focusing factor f_g is significantly larger than ours. For their choice of a mass $M = 5 \times 10^5\ M_\odot$ and a typical encounter velocity of 20 km/s, they give $f_g = 1.9$, whereas we arrive at $f_g \approx 1$. Napier and Staniucha's choice of efficiency factors therefore implies the half-life

$$t_{1/2}^{-1} = (1.8 \times 10^7\ \text{yr})^{-1} f_z f_e f_t f_s f_p f_g \left(\frac{\rho_0}{M_\odot/\text{pc}^3}\right)$$
$$= (1.8 \times 10^7\ \text{yr})^{-1} \times 1 \times 1 \times 1 \times 1 \times 1.5 \times 1.9 \times 0.027 = (2.3 \times 10^8\ \text{yr})^{-1}. \tag{48}$$

For comparison, Napier and Staniucha claim that the original comet population has been depleted by a factor of 1.4×10^{-3} over the age of the solar system. That number would imply a half-life for comet disruption of about 5×10^8 yr. Thus they *under*estimate the disruption rate by a factor of 2, given their assumptions. Their short lifetime arises entirely from their efficiency factors: compared with the values derived in the present paper, their choices for the six efficiency factors overestimate the rate of comet stripping by a factor of 10. The one-sided accumulation of all these effects has led them to overestimate the rate of comet disruption by over an order of magnitude, reducing the half-life from 2.7×10^9 to 2.3×10^8 yr.

Bailey (1983) has also investigated the disruption of the Oort cloud by encounters with MC's. He found that it was "unlikely" that the Oort cloud could survive for the age of the solar system. Bailey's equation (51) states that a comet at radius $r > r_0$ can escape in 4.5×10^9 yr, where

$$r_0 = \frac{1.2\times10^3\ \mathrm{AU}}{g_{\mathrm{tot}}^{1/3}}\left(\frac{1M_\odot/\mathrm{pc}^3}{\rho_0}\right)^{2/3}. \qquad (49)$$

Here ρ_0 is the density of MC's, and Bailey's g_{tot}, like our f_s, is equal to unity if there are no penetrating encounters. Our equation (5) can be cast into an analogous form (leaving out the efficiency factors other than f_s):

$$a = \frac{6.2\times10^2\ \mathrm{AU}}{f_s^{2/3}}\left(\frac{1M_\odot/\mathrm{pc}^3}{\rho_0}\right)^{2/3}\left(\frac{4.5\times10^9\ \mathrm{yr}}{t_{1/2}}\right)^{2/3}. \qquad (50)$$

Thus Bailey has *under*estimated the disruptive effect of the MC's by about a factor of 2 (in semimajor axis at fixed time) or 3 (in time at fixed semimajor axis). In contrast to Napier and Staniucha, Bailey corrects for the z motion of the Sun, using an efficiency factor $f_z = 0.4$, which is close to our value $f_z = 0.5$. However, he does not correct for the epicyclic motion of the Sun, and uses a finite-size factor f_s that is of order unity, rather than the value 0.5 that we prefer. These factors appear to account for the difference between Bailey's conclusion and our own.

V. WIDE BINARIES

A significant fraction of stars are found in wide binary systems with projected separations $\Delta r \approx 0.1$ pc $\approx 20\,000$ AU (Bahcall and Soneira 1981; Latham *et al.* 1984). These binaries are almost certainly primordial*; and their survival therefore provides an important constraint on the local disruption rate.

The half-life of wide binaries due to encounters with IC's is given by equation (5). Since the projected separations of the binary components are comparable to the semimajor axes of the comets in the Oort cloud, we shall evaluate (5) at the same semimajor axis $a = 25\,000$ AU that we used for the comets. However, the total mass of the binary is $m_1 + m_2 \approx 2\ M_\odot$ instead of 1 $M_\odot$ for a comet bound to the Sun. Thus, equation (7) becomes

$$t_{1/2}^{-1} = (2.5\times10^7\ \mathrm{yr})^{-1} f_z f_e f_t f_s f_p f_g \left(\frac{\rho_0}{M_\odot/\mathrm{pc}^3}\right). \qquad (51)$$

Most of the stars in the Latham *et al.* sample are G stars, whose rms z velocity is $\sigma_{z,\mathrm{G}}(t_0) = 20\ \mathrm{km\ s^{-1}}$. Using the model in Sec. III*a1*, their lifetime average z velocity is therefore $\langle\sigma_{z,\mathrm{G}}^2(t)\rangle^{1/2} = 14\ \mathrm{km\ s^{-1}}$ [Eq. (13)], and equations (10) and (11) yield $x = 2.4$ and $f_z = 0.28$.

To compute the epicycle factor, we average equation (17) over the G stars in the solar neighborhood and denote this average by $\langle\cdot\rangle_{\mathrm{G}}$. Thus $\langle v_\phi - V_c\rangle_{\mathrm{G}}$ is the asymmetric drift of G stars and is equal to $-9\ \mathrm{km\ s^{-1}}$ (Mihalas and Binney 1981, Table 6.3 and equation 6–30). The mean guiding center radius is therefore $\langle R_e\rangle_{\mathrm{G}} = R + \langle v_\phi - V_c\rangle_{\mathrm{G}}/2B = 9.6$ kpc. According to Sanders *et al.* (1984), the surface density of molecular hydrogen at 9.6 kpc is about 10% larger than at 10 kpc; hence we set the epicycle factor $f_e = 1.1$.

The other efficiency factors f_t, f_s, f_p, f_g are the same for wide binaries and comets. Thus our best estimate of the half-life of wide binaries is

$$t_{1/2}^{-1} = (2.5\times10^7\ \mathrm{yr})^{-1}\times0.28\times1.1\times1\times0.5\times1.5\times1\times0.024 = (4.5\times10^9\ \mathrm{yr})^{-1}, \qquad (52)$$

which is similar to the lifetime of a comet at the same semimajor axis, $t_{1/2} = 2.8\times10^9$ yr [Eq. (45)]. Since the mean age of the G stars in the solar neighborhood is about the same as the age of the Sun, the fraction of binaries disrupted should be comparable to the fraction of comets disrupted. Thus Bahcall and Soneira's observation that a substantial fraction of all stars are members of wide binaries provides strong evidence that most the Oort comet cloud has also survived. This result is completely independent of uncertainties in the density of molecular gas ρ_0 or the efficiency factor f_s, since these affect the half-lives of the comets and the binaries in exactly the same way.

VI. CONCLUSIONS

Our best estimates for the half-life of a comet with semimajor axis $a = 25\,000$ AU due to encounters with molecular and atomic clouds are 3×10^9 and 5×10^{10} yr, respectively. For comparison, the half-life due to encounters with field stars is 3×10^9 yr. The half-life due to field stars is relatively well determined, but the half-life due to molecular clouds is quite uncertain, for two main reasons:

(1) The conversion factor from CO intensity to H_2 column density, the mean density of molecular gas, the mean surface density of molecular clouds, and the clumpiness of the clouds are all imperfectly understood. Our half-life is based on a mean density $\rho_0 = 0.024\ M_\odot/\mathrm{pc}^3$, and a mean surface density and clumping factor [Eq. (31)] given by $N_{\mathrm{H}}C = 2\times10^{22}\ \mathrm{cm}^{-2}$.

(2) The temporal evolution of the local molecular-gas density is poorly known. The density evolves as a result of infall, star formation, stellar mass loss, and radial evolution of both the solar orbit and the molecular gas. We have adopted a mean density over the solar lifetime which is 1.5 times the present density.

The presence of a large number of wide binary stars with semimajor axes comparable to those in the Oort cloud provides an independent argument that the Oort cloud can survive the disruptive effects of encounters with interstellar clouds—or any other, as yet undiscovered, massive objects in the Galactic disk.

Although our conclusions are still tentative, we believe that the analysis developed in this paper forms a sound basis for more accurate evaluations of the cometary half-life as our understanding of the Galaxy improves.

*Note that the analogs of the two mechanisms that have been proposed for refilling the Oort comet cloud (capture of comets from interstellar space and scattering of comets out of a reservoir at smaller radii) cannot explain the existence of these systems. Capture of companion stars from interstellar space would require a three-body encounter, which occurs at a negligible rate; scattering of companion stars from smaller radii would yield a flat distribution in binding energy (Retterer and King 1982), which is inconsistent with the observed distribution of binary separations in the interval [0.01 pc, 0.1 pc] (Bahcall and Soneira 1981).

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APPENDIX

We investigate the half-life of a binary in the limit where the clouds are so diffuse that the maximum velocity perturbation received in a passage through a single cloud is much less than the escape velocity from the binary.

Let (x_1, x_2, x_3) be Cartesian coordinates in the rest frame of the cloud, oriented so that the velocity vector of the binary is $\mathbf{V} = V_r \hat{x}_3$. The potential $U(\mathbf{x})$ from the cloud satisfies Poisson's equation

$$\nabla^2 U = 4\pi G \rho. \tag{A1}$$

Assuming that $U(\mathbf{x})$ varies slowly on a scale equal to the binary separation $\Delta\mathbf{x}$, the relative acceleration of the binary components is $\Delta\mathbf{a}$, where

$$\Delta a_i = -\sum_{j=1}^{3} \frac{\partial^2 U}{\partial x_i \partial x_j} \Delta x_j. \tag{A2}$$

If we neglect the acceleration of the center of mass of the binary—which is a good approximation since the escape speed from a typical cloud is generally much less than the relative velocity V_r—then the center-of-mass trajectory is $x_1 = \text{constant}$, $x_2 = \text{constant}$, $x_3 = V_r t$. In the impulse approximation, the total velocity change is

$$\Delta v_i = -\frac{1}{V_r} \sum_{j=1}^{3} \Delta x_j \int_{-\infty}^{\infty} dx_3 \left(\frac{\partial^2 U}{\partial x_i \partial x_j} \right)_{x_1, x_2 = \text{const.}} . \tag{A3}$$

Since $\partial U / \partial x_j \to 0$ as $|x_3| \to \pm\infty$, we have

$$\Delta v_3 = 0,$$

$$\Delta v_i = \sum_{j=1}^{2} \Delta x_j \frac{\partial^2 B}{\partial x_i \partial x_j}, \quad i = 1,2, \tag{A4}$$

where

$$B(x_1, x_2) = -\frac{1}{V_r} \int_{-\infty}^{\infty} U(x_1, x_2, x_3)\, dx_3. \tag{A5}$$

Now average the total velocity change $\Delta v^2 = \Sigma_i \Delta v_i^2$ over orientation, writing $\langle \Delta x_i \Delta x_j \rangle = \frac{1}{3} r^2 \delta_{ij}$, where r is the binary separation. Thus

$$\langle \Delta v^2 \rangle = \frac{1}{3} r^2 \sum_{i=1}^{2} \sum_{j=1}^{2} \left(\frac{\partial^2 B}{\partial x_i \partial x_j} \right)^2. \tag{A6}$$

Next consider the average of $\langle \Delta v^2 \rangle$ over the area A in $x_1 x_2$ space, which is large compared to the projected area of the cloud, so that $B \to 0$ at the boundary of A. We have

$$\langle \Delta v^2 \rangle = \frac{1}{3} \frac{r^2}{A} \sum_{i,j=1}^{2} \int_A dx_1\, dx_2 \left(\frac{\partial^2 B}{\partial x_i \partial x_j} \right)^2$$

$$= \frac{1}{3} \frac{r^2}{A} \int_A dx_1\, dx_2 \left(\frac{\partial^2 B}{\partial x_1^2} + \frac{\partial^2 B}{\partial x_2^2} \right)^2, \tag{A7}$$

where the second line follows by integrating the term $(\partial^2 B / \partial x_1 \partial x_2)^2$ by parts. We may now write

$$\frac{\partial^2 B}{\partial x_1^2} + \frac{\partial^2 B}{\partial x_2^2} = -\frac{1}{V_r} \int_{-\infty}^{\infty} dx_3 \left(\nabla^2 U - \frac{\partial^2 U}{\partial x_3^2} \right); \tag{A8}$$

the contribution from the second term of the integrand vanishes, since $U \to 0$ as $|x_3| \to \infty$, and using Poisson's equation (A1), we have

$$\frac{\partial^2 B}{\partial x_1^2} + \frac{\partial^2 B}{\partial x_2^2} = -\frac{4\pi G \Sigma(x_1, x_2)}{V_r}, \tag{A9}$$

where Σ is the surface density of the cloud. The average value of r^2 is $\frac{7}{4} a^2$ for binaries which are uniformly distributed on the energy hypersurface in phase space, and thus

$$\langle \Delta v^2 \rangle = \frac{28\pi^2}{3} \frac{G^2}{V_r^2} \frac{a^2}{A} \int_A \Sigma^2 dx_1\, dx_2. \tag{A10}$$

The rate of encounters with clouds is nAV_r, where n is the number density of clouds, so the mean-square change in v in a time interval Δt is

$$\langle \Delta v^2 \rangle = \frac{28\pi^2}{3} \frac{G^2 n a^2 \Delta t}{V_r} \int_A \Sigma^2 dx_1 dx_2. \tag{A11}$$

At this point, we can define A to be the area of the cloud itself, since the surface density is zero outside the cloud. We next average over relative velocity using the result $V \langle 1/V_r \rangle = \sqrt{6/\pi}$, where V is the rms relative velocity. The total mass density in clouds is $\rho = n \int \Sigma\, dx_1\, dx_2$ and so we may write

$$\langle \Delta v^2 \rangle = \frac{7 \cdot 2^{5/2} \cdot \pi^{3/2}}{3^{1/2}} \frac{G^2 \rho a^2}{V} \Sigma_{av} C \Delta t, \tag{A12}$$

where the average surface density in the cloud is

$$\Sigma_{av} \equiv \frac{\int_A \Sigma\, dx_1\, dx_2}{\int_A dx_1\, dx_2} \tag{A13}$$

and the clumping factor is

$$C \equiv \frac{\int_A \Sigma^2\, dx_1\, dx_1 \int_A dx_1\, dx_2}{\left(\int_A \Sigma\, dx_1\, dx_2 \right)^2}. \tag{A14}$$

Having these results, we may now solve the Fokker-Planck equation, which governs the evolution of the binaries. Let $x = G(m_1 + m_2)/(2a)$ be the binding energy per unit mass and let $p(x,t)\, dx$ be the probability that a binary has binding energy in the range $x \to x + dx$ at time t. The Fokker–Planck equation for $p(x,t)$ reads

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x}(p x_1) + \frac{1}{2} \frac{\partial^2}{\partial x^2}(p x_2), \tag{A15}$$

where x_1 and x_2 are the mean and mean-square fluctuations in x per unit time. We have

$$x_1 = -\frac{1}{2} \frac{\langle \Delta v^2 \rangle}{\Delta t} = -\frac{K}{2x^2},$$

$$x_2 = \frac{\langle (\mathbf{v} \cdot \Delta \mathbf{v})^2 \rangle}{\Delta t} = \frac{1}{3} \langle v^2 \rangle \frac{\langle \Delta v^2 \rangle}{\Delta t} = \frac{2K}{3x}, \tag{A16}$$

where we have used the virial theorem $\langle v^2 \rangle = 2x$ and where

$$K = \frac{7 \cdot 2^{1/2} \pi^{3/2}}{3^{1/2}} \frac{G^4 \rho (m_1 + m_2)^2 \Sigma_{av} C}{V}. \tag{A17}$$

Equation (A15) now reads

$$\frac{\partial p}{\partial t} = K \left[\frac{1}{2} \frac{\partial}{\partial x} \left(\frac{p}{x^2} \right) + \frac{1}{3} \frac{\partial^2}{\partial x^2} \left(\frac{p}{x} \right) \right]. \tag{A18}$$

The Green's function $p_G(x,t)$ defined by $p_G(x, t=0)$

$= \delta(x - x_0)$ is

$$p_G(x,t) = \frac{x_0^{5/4} x^{3/4}}{Kt} \exp\left(-\frac{x_0^3 + x^3}{3Kt}\right) I_{5/6}\left[\frac{2(xx_0)^{3/2}}{3Kt}\right], \tag{A19}$$

where I denotes a modified Bessel function. The fraction of binary stars remaining after time t is

$$f(t) = \int_0^\infty p_G(x,t)\, dx$$

$$= \frac{\gamma(\frac{5}{6}, x_0^3/3Kt)}{\Gamma(\frac{5}{6})}, \tag{A20}$$

where γ and Γ denote the incomplete and complete gamma functions.* The half-life is defined by $f(t_{1/2}) = \frac{1}{2}$, which occurs when $x_0^3/3Kt = 0.5330$, and hence

$$t_{1/2} = \frac{V(m_1 + m_2)}{0.5330 \cdot 56 \cdot 6^{1/2} \cdot \pi^{3/2} G\rho a^3 \Sigma_{av} C} \tag{A21}$$

$$= 0.0025 \frac{V(m_1 + m_2)}{G\rho a^3 \Sigma_{av} C},$$

the result quoted in Equation (30).

*After this work was complete, we learned that Bailey (1985) has independently derived equations (A19) and (A20), although with a different formula for the diffusion coefficient K.

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